

Kinetic dust acoustic mode in inhomogeneous partially magnetized plasma

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2014 J. Phys.: Conf. Ser. 511 012011

(<http://iopscience.iop.org/1742-6596/511/1/012011>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 134.58.253.57

This content was downloaded on 08/05/2014 at 08:16

Please note that [terms and conditions apply](#).

Kinetic dust acoustic mode in inhomogeneous partially magnetized plasma

J. Vranjes and S. Poedts

Center for Plasma Astrophysics,
and Leuven Mathematical Modeling and Computational Science Research Centre (LMCC),
Celestijnenlaan 200B, 3001 Leuven, Belgium.

E-mail: jvranjes@yahoo.com; Stefaan.Poedts@wis.kuleuven.be

Abstract. The dust acoustic mode in inhomogeneous plasmas is discussed in the regime when the electrons and ions are magnetized while at the same time the dust grains remain un-magnetized. Although the dynamics of the light species is strongly affected by the magnetic field, the dust acoustic mode may still propagate in practically any direction. The inhomogeneity implies a source of free energy for an instability that develops through the diamagnetic drift effects of the magnetized species. It is shown that this may be a powerful mechanism for the excitation of dust acoustic waves. The analysis presented in this work is also applicable to plasmas containing both positive and negative ions and electrons, provided that at least one of the two ion species is un-magnetized.

1. Introduction

Inhomogeneous plasmas imply the presence of a source of free energy that can make some plasma modes growing. In an electron-ion plasma, when both electrons and ions are magnetized, the corresponding growing low-frequency mode is the drift mode. However, in some situations, in the perturbed state the heavier species (ions) may in fact behave as un-magnetized regardless of the presence of a magnetic field. In view of the mass difference, for electrons we may have at the same time $\omega \ll \Omega_e = eB_0/m_e$. Electrostatic modes propagating in a plasma with such properties will have the basic features of the ion acoustic (IA) mode, in spite of the magnetized electrons. Within the two-fluid theory such a mode in an inhomogeneous plasma may become growing [1] provided the simultaneous presence of collisions and the mentioned equilibrium density gradient perpendicular to \vec{B}_0 . Within the kinetic theory the mode is also growing due to purely kinetic effects. The growth rate is similar to the standard drift wave instability [2], with the necessary condition that the wave frequency is below the electron diamagnetic drift frequency $\omega_{*e} = v_{*e}k_{\perp}$.

In a plasma containing also some dust grains, and in the presence of some small perturbations, the time and space scales at which the grain dynamics develops is orders of magnitude different as compared to the above mentioned drift and IA wave scales. Hence, both the ions and electrons may be well magnetized, while the dust grains may not feel the effects of the magnetic field. One may then expect that the grain dynamics remains within the dust acoustic (DA) wave description although the lighter species are magnetized and thus have very different dynamics in the directions parallel and perpendicular to the magnetic field vector. In an inhomogeneous plasma, the dynamics of the electrons and ions will be described by a set of equations that follow from the standard drift wave theory, with possibly the only difference regarding the finite



ion inertia effects that at the DA wave scale can simply be ignored. The resulting dispersion equation may then yield a growing DA wave, with the instability driven by the density gradient of the magnetized light particles. This type of the dust acoustic, gradient-driven instability will be studied in the present work. In particular, we shall use a model in which the equilibrium dusty plasma is confined by the magnetic field, implying that all species are magnetized, while the perturbations will be studied. Two specific cases will be studied, viz. i) an electron-ion plasma with negatively charged grains, and ii) a plasma containing positively charged grains and negative ions and electrons.

2. Derivations and results

We shall assume a plasma placed in an external magnetic field $\vec{B}_0 = B_0 \vec{e}_z$, with a density gradient of the species j in the x -direction ($j = e, i, d$). The charge number Z_d of the dust grains is assumed to be constant (which is appropriate for frequencies far below the mean attachment frequencies for electrons and ions so that the equilibrium quasi-neutrality condition reads $n_{i0}(x) = n_{e0}(x) + Z_d n_{d0}(x)$). A varying grain charge is known to introduce an extra mode of oscillations [3]. In such a geometry, the species will have the above described equilibrium diamagnetic drift velocities $\vec{v}_{*j} = \vec{e}_z \times \nabla p_j / (q_j n_j B_0)$. The perturbed number density of the species j is $n_{j1} = \int_{-\infty}^{\infty} f_{j1} d^3 \vec{v}$, where f_{j1} is the corresponding distribution function. In the perturbed state, we shall continue to treat the light species as magnetized. However, the dust grains are taken as un-magnetized, implying wave-lengths that are above the ion and electron gyro-radii, yet below the grain gyro-radius, or/and the wave frequencies that are below the electron and ion gyro-frequency, but above the grain gyro-frequency. Hence, the derivation will be different for magnetized and un-magnetized species. The former yields for the perturbed number density

$$n_{j1} = -\frac{q_j n_{j0} \phi_1}{\kappa T_j} \left\{ 1 + (\omega - \omega_{*j}) \sum_m \frac{\Lambda_m(\beta_j)}{\omega - m\Omega_j} \left[W \left(\frac{\omega - m\Omega_j}{|k_z| v_{Tj}} \right) - 1 \right] \right\}. \quad (1)$$

Here, $\beta_j = k_{\perp}^2 \rho_j^2$, $\Lambda_m(\beta_j) = I_m(\beta_j) \exp(-\beta_j)$, $W(\chi) = (2\pi)^{1/2} \int_{-\infty}^{+\infty} \eta \exp(-\eta^2/2) d\eta / (\eta - \chi)$, I_m is the modified Bessel function of the first kind and order m . We use the local approximation with small perturbations of the form $\sim \hat{f}(x) \exp(-i\omega t + ik_y y + ik_z z)$; $\hat{f}(x)$ is the x -dependent amplitude and $|d/dx| \ll |k_y|$. For frequencies much below Ω_j , we keep only the term $m = 0$ and, being interested in the DA spatial and time scales, it is good enough to use $\Lambda_0(\beta_j) \simeq 1$ for both electrons and ions. For similar reasons ($|\chi| \ll 1$) we shall use the approximate expression $W(\chi) \simeq 1 - \chi^2 + \dots + i(\pi/2)^{1/2} \chi \exp(-\chi^2/2)$. This yields for electrons and ions

$$\frac{n_{j1}}{n_{j0}} = -\frac{q_j \phi_1}{\kappa T_j} \left[1 + i \left(\frac{\pi}{2} \right)^{1/2} \frac{\omega - \omega_{*j}}{|k_z| v_{Tj}} \exp \left(\frac{\omega^2}{2k_z^2 v_{Tj}^2} \right) \right]. \quad (2)$$

Regarding the perturbed dust dynamics, we shall assume frequencies satisfying the conditions $\omega \gg \Omega_d$, and $\omega \gg \omega_{*d}$. The latter is particularly easily satisfied for grains with a high charge number. Further in the text, these assumptions will be supported by the parameters which we use. In such a frequency limit for dust grains we have

$$\frac{n_{d1}}{n_{d0}} = \frac{e Z_d \phi_1}{\kappa T_d} \left[1 - \mathcal{Z} \left(\frac{\omega}{k v_{Td}} \right) \right]. \quad (3)$$

Here, $\mathcal{Z}(\alpha) = [\alpha/(2\pi)^{1/2}] \int_{-\infty}^{+\infty} d\xi \exp(-\xi^2/2) / (\alpha - \xi)$ is the plasma dispersion function. Note that the argument α here comprises the total k , which describes the essential fact that the grain perturbations are independent on the direction of the magnetic field vector $\vec{B}_0 = B_0 \vec{e}_z$.

After expanding $\mathcal{Z}(\alpha) \simeq 1 + 1/\alpha^2 + \dots - i(\pi/2)^{1/2}\alpha \exp(-\alpha^2/2)$, what is valid if $|\alpha| \gg 1$ and $|Re(\alpha)| \gg |Im(\alpha)|$, and using Eqs. (2) and (3), the quasi-neutrality condition, which in the present case reads $n_{i1} = n_{e1} + Z_d n_{d1}$, yields the dispersion equation:

$$\Delta(\omega, \vec{k}) \equiv \frac{k^2 Z_d^2 n_{d0}}{m_d \omega^2} - \frac{n_{i0}}{\kappa T_i} - \frac{n_{e0}}{\kappa T_e} - i \left(\frac{\pi}{2} \right)^{1/2} \left[\frac{\omega}{k v_{Td}} \frac{Z_d^2 n_{d0}}{\kappa T_d} \exp \left(-\frac{\omega^2}{2k_z^2 v_{Td}^2} \right) + \frac{(\omega - \omega_{*i})}{k_z v_{Ti}} \frac{n_{i0}}{\kappa T_i} \exp \left(-\frac{\omega^2}{2k_z^2 v_{Ti}^2} \right) + \frac{(\omega - \omega_{*e})}{k_z v_{Te}} \frac{n_{e0}}{\kappa T_e} \exp \left(-\frac{\omega^2}{2k_z^2 v_{Te}^2} \right) \right] = 0. \quad (4)$$

The real part of Eq. (4) yields the DA wave frequency $\omega_r^2 = k^2 c_d^2$, $c_d^2 = \kappa T_e T_i n_{d0} Z_d^2 / [m_d (n_{i0} T_e + n_{e0} T_i)]$. The growth rate (and damping) γ is given approximately by $\gamma \simeq -Im(\Delta) / [\partial Re(\Delta) / \partial \omega]_{\omega=\omega_r}$, and this yields

$$\gamma = - \left(\frac{\pi}{2} \right)^{1/2} \frac{m_d c_d^3 k}{2 Z_d^2 n_{d0}} \left[\frac{\omega_r n_{d0} Z_d^2}{k v_{Td} \kappa T_d} \exp \left(-\frac{\omega_r^2}{2k_z^2 v_{Td}^2} \right) + \frac{(\omega_r - \omega_{*i})}{k_z v_{Ti}} \frac{n_{i0}}{\kappa T_i} \exp \left(-\frac{\omega_r^2}{2k_z^2 v_{Ti}^2} \right) + \frac{(\omega_r - \omega_{*e})}{k_z v_{Te}} \frac{n_{e0}}{\kappa T_e} \exp \left(-\frac{\omega_r^2}{2k_z^2 v_{Te}^2} \right) \right]. \quad (5)$$

The diamagnetic drift separates charges so that in the present case ω_{*i} has the sign opposite to ω_{*e} . The shape of Eq. (5) reveals that, for a positive ω_{*e} , the only term capable of changing the sign of γ and yielding the growth is in principle the electron term, provided that $\omega_r < \omega_{*e}$. The dust term in Eq. (5) is typically always negligible in comparison to the others. To check if the instability is really possible, we make the ratio of the ion and electron terms γ_i/γ_e that are within the brackets [] in Eq. (5). As long as $\omega_r^2 \ll k_z^2 v_{Te}^2, k_z^2 v_{Ti}^2$, the exponential terms are practically equal to unity and play no role. Also, having $\omega_r \ll \omega_{*e}, \omega_{*i}$, it is seen that the instability may take place only provided that

$$|\gamma_i/\gamma_e| \simeq \left| \frac{n'_{i0}}{n'_{e0}} \right| \left(\frac{T_e m_i}{T_i m_e} \right)^{1/2} = \left| 1 + \frac{Z_d n'_{d0}}{n'_{e0}} \right| \left(\frac{T_e m_i}{T_i m_e} \right)^{1/2} < 1. \quad (6)$$

Hence, here the instability implies unrealistically high values of the ion temperature and the excitation of DA waves by electron terms in such dusty plasmas containing negatively charged grains is unlikely. An exception could be a specific case in which the electron and dust density gradients have opposite signs, and, in addition, nearly balance each other so that the term $|1 + Z_d n'_{d0}/n'_{e0}| \simeq |1 - Z_d n_{d0} L_e / (n_{e0} L_d)|$ becomes very small, implying a nearly flat ion density profile, i.e., $L_e/L_i \ll 1$. Note that in standard electron-ion plasma (e.g., in the case of the drift wave) exactly the same γ_e term appears, resulting in the instability mentioned in the introductory lines, while in the same time it is usually $\omega_r^2 > k_z^2 v_{Ti}^2$, or $\omega_r^2 \gg k_z^2 v_{Ti}^2$. The total ion contribution is thus considerably reduced and the electron terms are capable of making the mode growing. The reason for the absence of the electron-driven instability in the present dusty plasma case is clearly only due to the fact that the frequencies are low and the exponential term in the ion part plays no role, i.e., $\omega_r^2 \ll k_z^2 v_{Ti}^2$.

Under typical laboratory conditions in the past, grains are negatively charged by electron attachment and this corresponds to the case discussed above. However, more recent experiments dealing with the photoelectric charging [4], and the presence of negative ions in a dusty plasma yield positively charged dust grains. In space and astrophysical plasmas, grains can have both

positive and negative charges, sometimes even at the same time, or in the form of neighboring layers containing opposite charges on the grains. This peculiar behavior of a dusty plasma is due to several competing charging mechanisms, like the attachment of plasma particles due to inelastic collisions, secondary electron emission and photoemission. Dusty plasmas containing negative ions imply a reduced charging of the grains due to electron attachment and dust grains can become positively charged, this especially in the case when positive ions are much lighter than negative ones. In such cases the diamagnetic velocities of electrons and any other negatively charged magnetized species have the same sign, and therefore their destabilizing effects will add up.

In view of that, performing a similar procedure as above, it can easily be shown that such plasmas are additionally destabilized, with the effects of negatively charged magnetized species (ions or grains) being similar to those of electrons. In the equilibrium we have $Z_d n_{d0}(x) = n_{i0}(x) + n_{e0}(x)$. Here, the density gradients of electrons and negative ions are assumed to be of the same sign, and they are balanced by the variation of the density of positively charged grains. In the perturbed state the dispersion equation is obtained from $Z_d n_{d1} = n_{i1} + n_{e1}$. The real part of the dispersion equation will again yield the same frequency, while the growth rate will be similar as Eq. (5), except for the ion contribution that now contains the negative sign in the critical term $\omega - \omega_{*i}$. If the dust term in Eq. (5) is negligible, it is seen that as long as $T_i \leq T_e$ the dust-acoustic mode will always be growing whenever $\omega_r < \omega_{*i}$. In case of plasmas with hot negatively charged magnetized ions $T_i \geq T_e$, the instability becomes determined by the electron term. The instability in the present case follows from the fact that both electrons and singly charged negative ions together change the sign of the growth rate γ .

In Fig. 1, we present the growth rate for positively charged grains and negative ions for an arbitrary set of plasma parameters, and in terms of the angle of propagation $\theta = \arctan(k_z/k_y)$ between the wave-vector \vec{k} and the y -axis. The electron and negative-ion number densities are taken equal to $n_{i0} = n_{e0} = n_0 = 10^{18} \text{ m}^{-3}$, and we take $Z_d = 10^3$ yielding $n_{d0} = 2 \cdot 10^{15} \text{ m}^{-3}$. We assume m_i to be equal to proton mass, for grains we take $m_d = 10^{-16} \text{ kg}$, and we assume a plasma with $L \equiv L_e = L_i = (d \log n_0 / dx)^{-1} = 1 \text{ m}$. For the wave-number we take $k = 10^3 \text{ m}^{-1}$ so that $L/\lambda = 160$, $\lambda = 2\pi/k$, and $\rho_d/\lambda = 3$, where $\rho_d = v_{Td}/\Omega_d$ and the magnetic field is taken as $B_0 = 0.2 \text{ T}$.

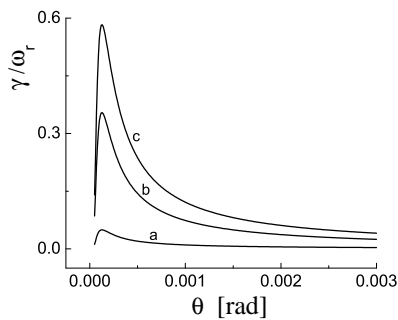


Figure 1. The normalized growth rate γ/ω_r in terms of the angle between \vec{k} and the y -axis, for $T_e = T_i = 0.1, 0.5, 1 \text{ eV}$ (lines a, b, c respectively).

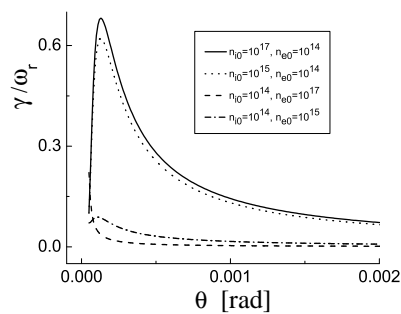


Figure 2. The normalized growth rate γ/ω_r in terms of the angle between \vec{k} and the y -axis, for several values n_{i0} and n_{e0} , and for $n_{d0} = (n_{i0} + n_{e0})/Z_d$.

The lines a, b, c in Fig. 1 correspond to three different values of the electron and ion temperature $T_e = T_i = T = 0.1, 0.5, 1 \text{ eV}$, respectively, where in the same time $T_d = T/50$.

The growth rate is strongly dependent on both the angle of propagation and the temperature of the light plasma species that are responsible for the growth of the mode. Note that ω_r here takes values between 400 and 1260 Hz while $\Omega_d = 0.3$ Hz, and ω_{*d} changes between 0.01 Hz and 0.1 Hz, so that indeed we have properly assumed limits for the local approximation, unmagnetized grains, and negligible dust diamagnetic drift effects.

We stress that taking any other value for the dust temperature, even going to the limit $T_d \sim T$, yields exactly the same shape for the growth rate. This is because for these parameters the dust contribution in Eq. (5) is completely negligible. Similarly, the growth rate does not change at all by varying n_0 , and this holds as long as n_{i0}, n_{e0} are kept equal. This may be expected from Eq. (5) because the density n_0 cancels out by the dust term $Z_d n_{d0}$ that is in the denominator in front of the brackets. However, for any other values $n_{i0} \neq n_{e0}$ the growth rate changes. This is presented in Fig. 2 for several number densities n_{i0}, n_{e0} (per cubic meter) and for the case $T = 0.5$ eV, $T_d = T/50$. Here, the corresponding dust density is given by $Z_d n_{d0} = n_{i0} + n_{e0}$.

The described mechanism of excitation of dust acoustic waves in inhomogeneous plasmas with magnetized light species is very likely to occur in any inhomogeneous environment. In such situations, the DA wave will practically always be growing and the growth-rate can easily be far above the wave frequency. The mode is most easily excited in a very narrow angle around a direction that is almost perpendicular to the magnetic field vector and the density gradient. In astrophysical clouds the proposed instability may become an initial triggering mechanism for the fragmentation, especially if self-gravity effects are also taken into account. The kinetic analysis presented here is in fact the most appropriate for such astrophysical plasmas with a small amount of collisions. Artificially produced grains under laboratory conditions in the recent past are also positively charged [4, 5, 6], and the proposed instability discussed here will undoubtedly work there too. However, the plasmas in these recent experiments include positively charged grains, electrons and negative ions, but, in addition to this, also a variable amount of positive ions [5], or negative grains [4]. The simplified model with three species only, discussed in the text, can easily be generalized to include the effects of these additional species. In both cases presented here it is in fact the ion density gradient that crucially determines the mode behavior.

Acknowledgments

The results presented here are obtained in the framework of the projects G.0304.07 (FWO-Vlaanderen), C 90347 (Prodex), GOA/2009-009 (K.U.Leuven). Financial support by the European Commission through the SOLAIRE Network (MTRN-CT-2006-035484) is gratefully acknowledged.

- [1] Vranjes J and Poedts S 2009 *Phys. Plasmas* **16** 022101
- [2] Krall N. A. and Book D 1969 *Phys. Rev. Lett.* **23** 574
- [3] Vranjes J, Pandey B P and Poedts S 2001 *Phys. Rev. E* **64** 066404
- [4] Sikafoose A. A., Coldwell J. E., Horányi M and Robertson S 2000 *Phys. Rev. Lett.* **84** 6034
- [5] Merlino R. L. and Sun-Hyun Kim 2006 *Appl. Phys. Lett.* **89** 091501
- [6] Rosenberg M and Merlino R. L. 2007 *Planet. Space Sci.* **55** 1464